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Electromagnetic Scattering Models for GNSS-R Land Applications Including Effects of Multiple Elevations in Random Rough Surfaces

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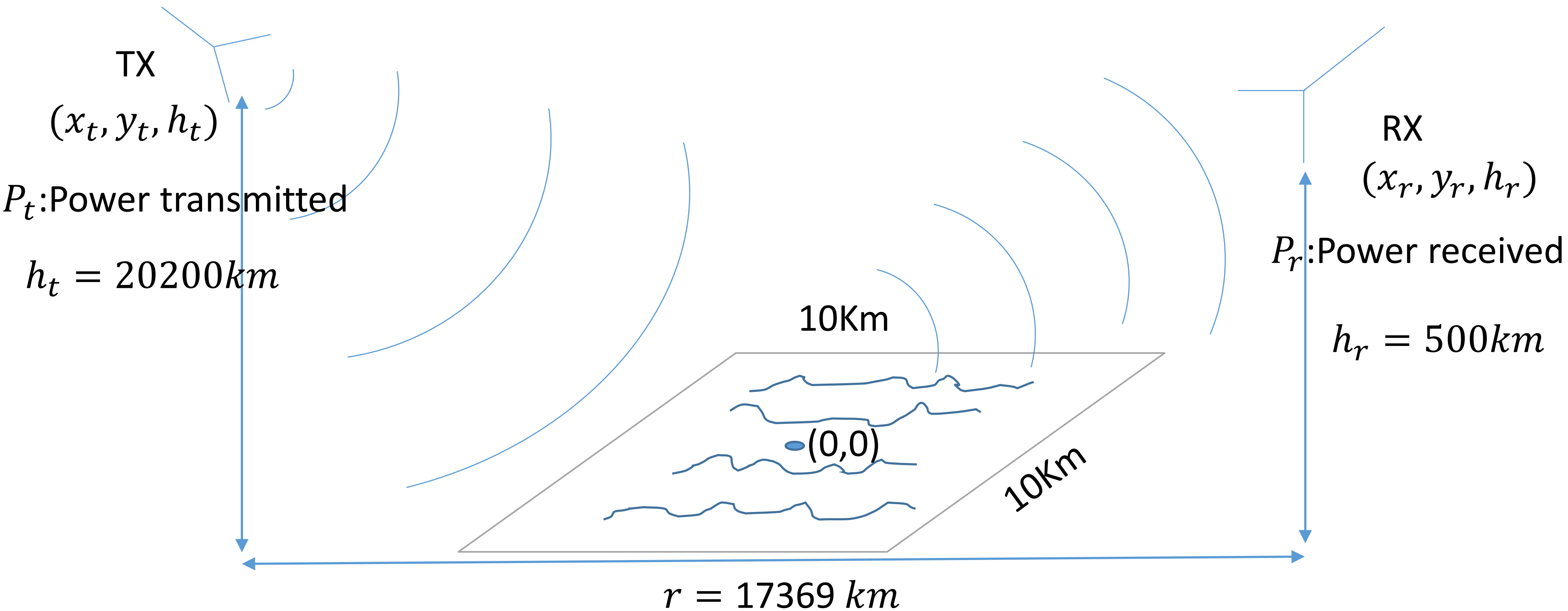
Outline

- GNSS-R physical problem
  - Land surface with multiple elevations
- Common models
  - Coherent model
  - Incoherent model
- Two recent models by our Group
  - Numerical Kirchhoff Simulator (KA, 2cm by 2cm patch)
  - Patch model with Numerical Solutions of Maxwell equations in 3D (NMM3D) (30m by 30m patch)

GNSS-R geometry

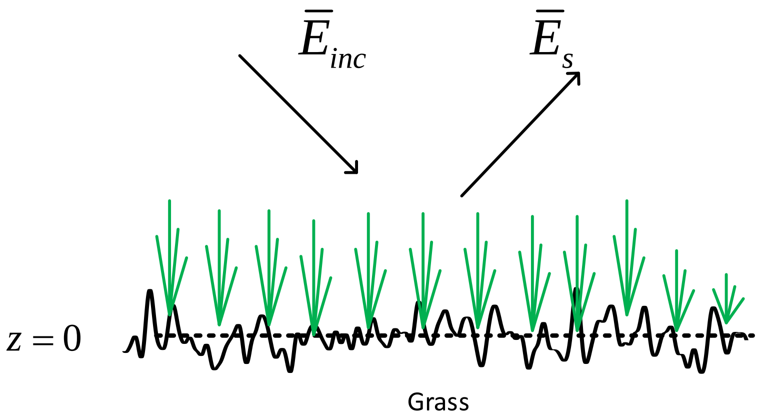


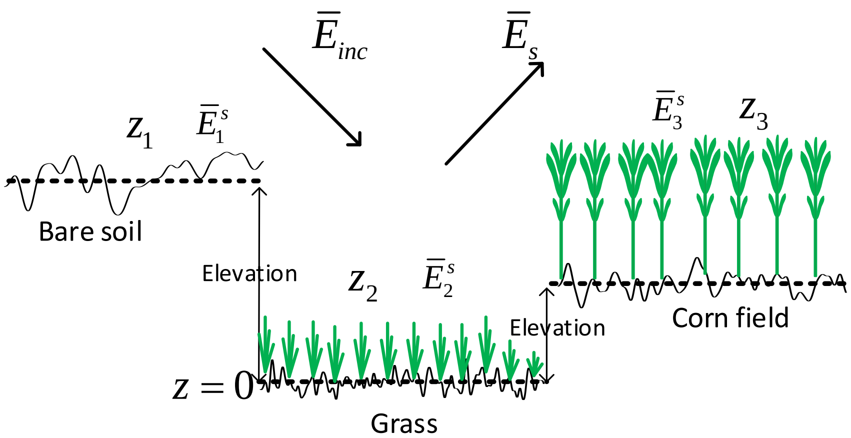
- Specular point (0,0)
- Calculate received power ratio Pr/Pt
- Area: 10km by 10km

Land surfaces: single elevation vs. multiple elevations

Single elevation (usual rough surface problem)

Multiple elevations (real land surfaces)





- ❑ Electromagnetics: rough surface specular scattering with multiple elevations
- ❑ Multiple elevations: digital elevation model (DEM) , e.g. 30m by 30m

Common Models: Coherent and Incoherent Model, large differences

- **"Coherent" model, assume single elevation :**

$$\frac{P_r}{P_t} = \frac{G_t}{4\pi(R_t + R_r)^2} \frac{G_r \lambda^2}{4\pi} \Gamma e^{-4k^2 h^2 (cos\theta)^2}$$

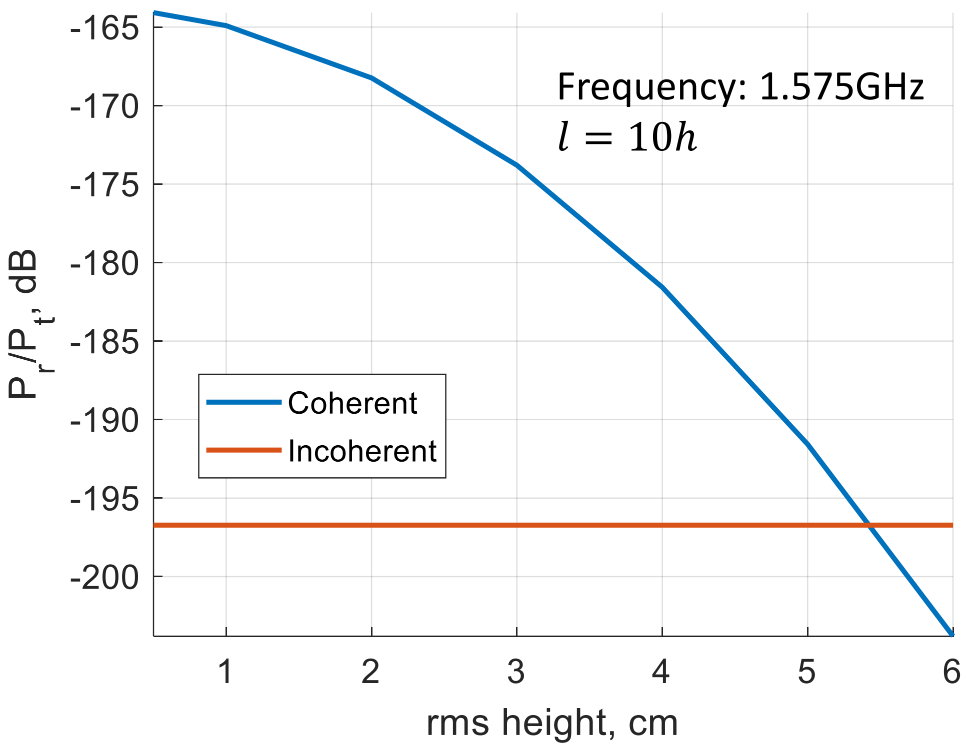
- **"Incoherent" model:**

$$\frac{P_r}{P_t} = \frac{G_t}{4\pi R_t^2} \frac{1}{4\pi R_r^2} \frac{G_r \lambda^2}{4\pi} \int \gamma \, dA$$

$\gamma = \frac{|\Gamma|}{2s^2}$: Bistatic coeff.

$s = \frac{\sqrt{2}h}{l}$: slope, Gaussian

$h: rms\ height$
$l: Correlation\ length$



- Up to 35 dB differences

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Surface profiles of multiple elevations

- ❑ Height function $f(x,y)$

$$f(x,y) = f_r(x,y) + f_{DEM}(x,y)$$

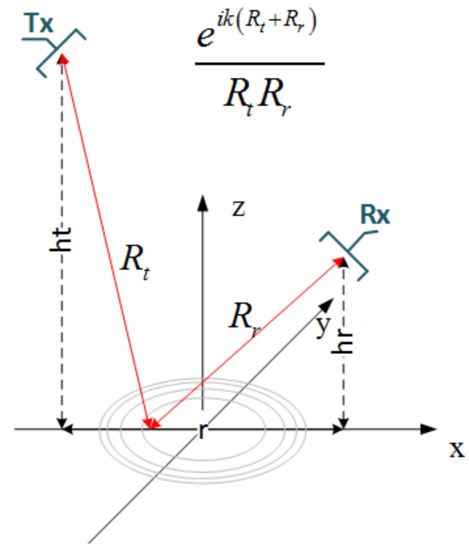
$f_r(x,y)$ = microwave centimeter roughness
f_{DEM}=elevations, tens of meters

- ❑ Multiple elevations cause phase variations (influences on coherent waves)

- ❑ Consider both coherent fields and incoherent fields

Kirchhoff Numerical Simulator (KA simulator)

$$\bar{E}_s(\bar{r}) = \frac{ik}{4\pi}\sqrt{\frac{P_t\eta_0}{2\pi}}\int\int dxdy\,\frac{e^{ik(R_t+R_r)}}{R_tR_r}\left(\bar{\bar{I}} - \hat{k}_s\hat{k}_s\right)\cdot\bar{F}(\alpha,\beta)$$



- ❑ $e^{ik(R_t+R_r)}$: phase variations of spherical waves and multiple elevations

- ❑ $\int\int dxdy = Area = 10km \times 10km$
  account for phase variations
  $patch:\ \Delta x\Delta y = 2cm \times 2cm$:
  $\mathrm{N} = \left(\frac{10^4}{0.02}\right)\times\left(\frac{10^4}{0.02}\right) = 2.5\times10^{11}$ patches
- ❑ Parallel implementation: 40 hours (20 cores)
- ❑ Brute force: keep track of phase of coherent wave

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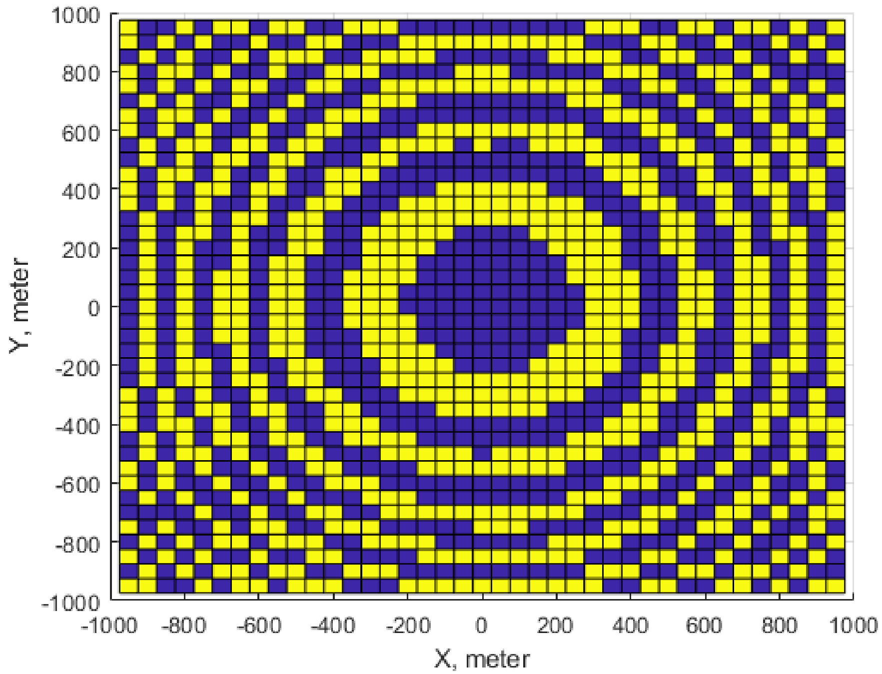
W. Gu, H. Xu and L. Tsang, "A numerical Kirchhoff simulator for GNSS land applications," Progress in electromagnetics research, vol. 164, pp119-133, 2019.

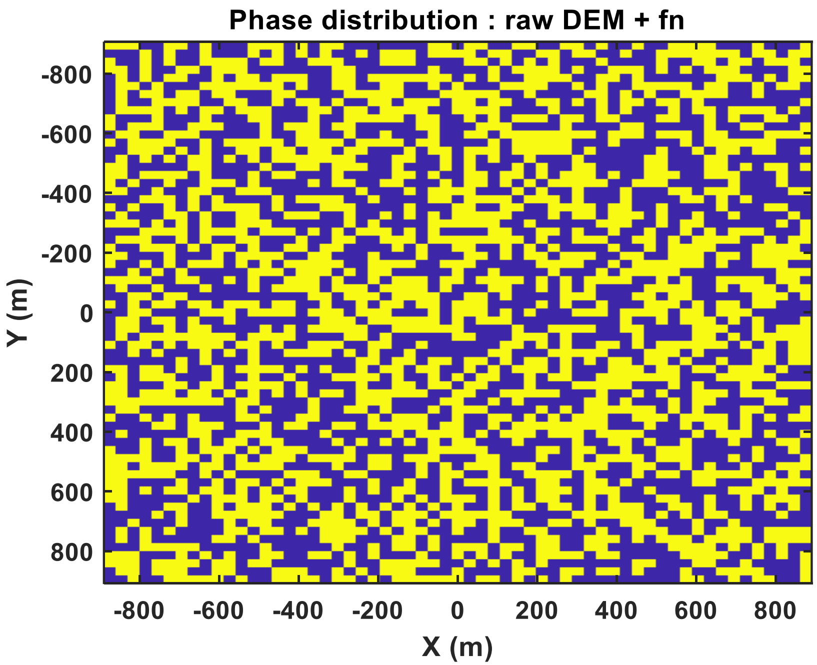
KA simulator: phase distribution

- ❑ Phase: every 2cmX2cm patch
- ❑ Blue and yellow alternate phase by π

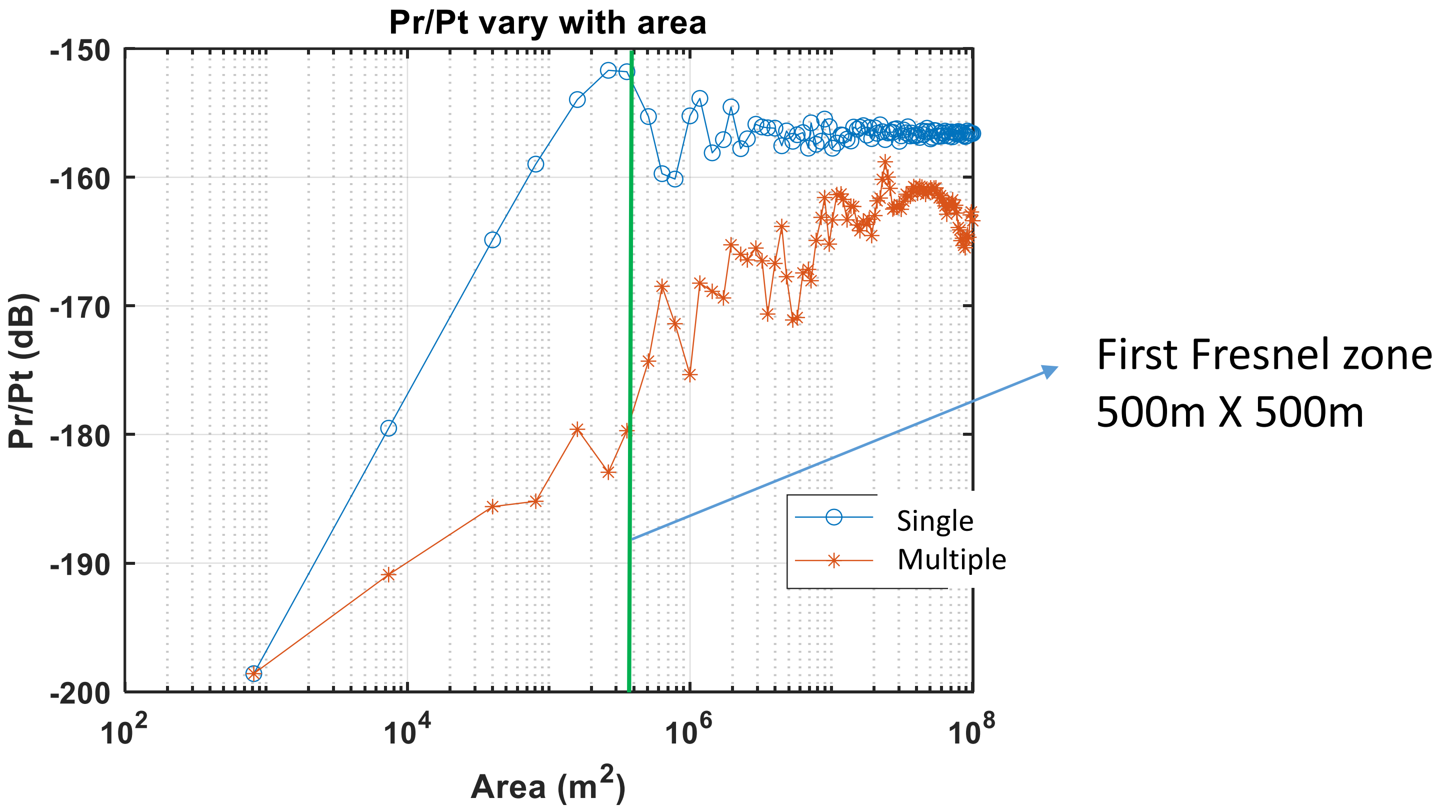
Single elevation:
Fresnel zones exhibited



Multiple elevations (DEM):
Fresnel zones disappear



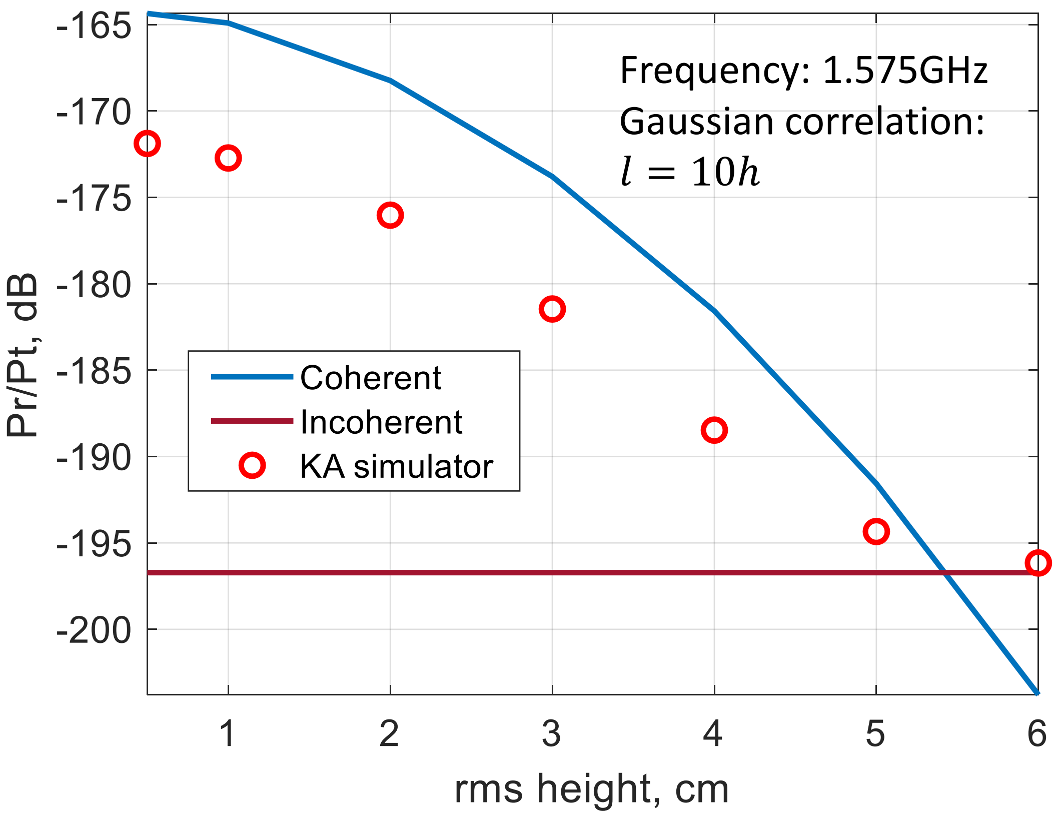
KA simulator: contributions by area



- ❑ Single elevation: power from first Fresnel zone
- ❑ Multiple elevations: first Fresnel zone, only 5% of power

KA simulator: multiple elevations (90000 elevations)

Multiple elevations in Georgia
31°49′50″N, 83°49′50″W



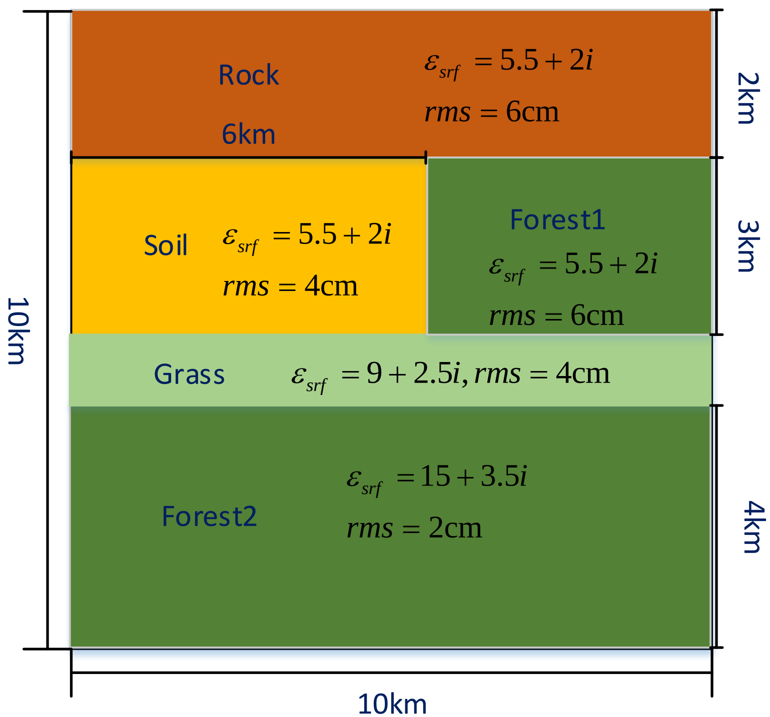
DEM resolution 30m by 30m

- ❑ Larger than incoherent
- ❑ smaller than the coherent model

Recent Patch Model: Motivations

	KA simulator	Patch model
Patch size	2cm by 2cm	30m by 30m
Computation requirements	40 hours (20 cores)	0.6 seconds (1 core)
Accuracy	Kirchhoff approximation	Numerical solutions of Maxwell Equations LUT (Accurate)
Surface type	Only Gaussian	Gaussian and exponential

Patch Model: 30m by 30m patches



- ❑ A land surface with multiple elevations
- ❑ Divided into physical areas based on surface properties, e.g. bare soil, grass, forests
- ❑ Each physical area is discretized into patches with size of 30m by 30m

Formulation: Correlation Formula

- Total scattered field = sum of scattered fields of N physical areas

$$\bar{E}_s = \sum_{n=1}^{N} \bar{E}_n^s, \ \bar{E}_n^s \text{ scattered field of nth area}$$

- Absolute value squared to get scattered power

Correlation formula

$$\left\langle \left| \bar{E}_s \right|^2 \right\rangle = \sum_{n=1}^{N} \left\langle \left| \bar{E}_n^s \right|^2 \right\rangle + \sum_{n=1}^{N} \sum_{m=n+1}^{N} 2\,\mathrm{Re}\left(\left\langle \bar{E}_n^s \right\rangle \left\langle \bar{E}_m^{s*} \right\rangle \right)$$

- $\langle|\bar{E}_n^s|^2\rangle$: power of nth physical area= coherent power + incoherent power
- $\langle\bar{E}_n^s\rangle\langle\bar{E}_m^{s*}\rangle$: correlations of different physical areas. Only include correlations of coherent fields from different physical areas

Equivalent formula: Coherent + Incoherent

Total scattered power = coherent contributions
+ incoherent contributions

Coherent & incoherent formula

$$\left\langle \left|\bar{E}_s\right|^2 \right\rangle = \left|\left\langle \bar{E}_s \right\rangle\right|^2 + \sum_{n=1}^{N} \left\langle \left|\bar{E}_n^s - \left\langle \bar{E}_n^s \right\rangle\right|^2 \right\rangle$$

- ❑ Net coherent field $\langle \bar{E}_s \rangle$: complex sum of coherent fields from N areas

$$\langle \bar{E}_s \rangle = \sum_{n=1}^{N} \langle \bar{E}_n^s \rangle, \langle \bar{E}_n^s \rangle \; coherent\ field\ of\ nth\ area$$

- ❑ Incoherent contributions: sum of incoherent intensities from N areas

$$incoherent\ power = \sum_{n=1}^{N} \langle |\bar{E}_n^s - \langle \bar{E}_n^s \rangle|^2 \rangle$$

- ❑ Equivalent to correlation formula

NMM3D (Maxwell Equations) implementation: calculate coherent and incoherent field of each area

- $\langle \bar{K}(\hat{k}_i, \hat{k}_s)\rangle$: coherent field of each patch (30m by 30m)
- $\left|(\hat{k}_i, \hat{k}_s) - \langle \bar{K}(\hat{k}_i, \hat{k}_s)\rangle\right|^2$: Incoherent Intensity of each patch
- NMM3D: compute both, Look up table (LUT)
  1. Rough surfaces
  2. Vegetation/forests
- Coherent field of nth area: coherent addtion

$$\left\langle \bar{E}_n^s(\bar{r})\right\rangle = \sqrt{\frac{P_t\eta}{2\pi}} \iint_{n\text{th area}} \frac{dxdy}{L_x L_y} \frac{\operatorname{sinc}(k_{dx}L_x/2)\operatorname{sinc}(k_{dy}L_y/2)}{R_t R_r} \exp\left(ik\left(R_r + R_t\right)\right)\left\langle \bar{K}\left(\hat{k}_i, \hat{k}_s\right)\right\rangle$$

  - $\exp\left(ik(R_r + R_t)\right)$: phase change of spherical wave and elevation change
  - $\operatorname{sinc}(\frac{k_{dx}L_x}{2})\operatorname{sinc}(\frac{k_{dy}L_y}{2})$: peak in specular direction of coherent field
  - L_x and L_y: patch size, 30m by 30m
- Incoherent : incoherent addition

$$\left\langle \left|\bar{E}_n^s(\bar{r}) - \left\langle \bar{E}_n^s(\bar{r})\right\rangle\right|^2\right\rangle = \frac{P_t\eta}{2\pi} \iint_{n\text{th area}} \frac{dxdy}{L_x L_y}\left(\frac{1}{R_t R_r}\right)^2 \left\langle \left|\bar{K}\left(\hat{k}_i, \hat{k}_s\right) - \left\langle \bar{K}\left(\hat{k}_i, \hat{k}_s\right)\right\rangle\right|^2\right\rangle$$

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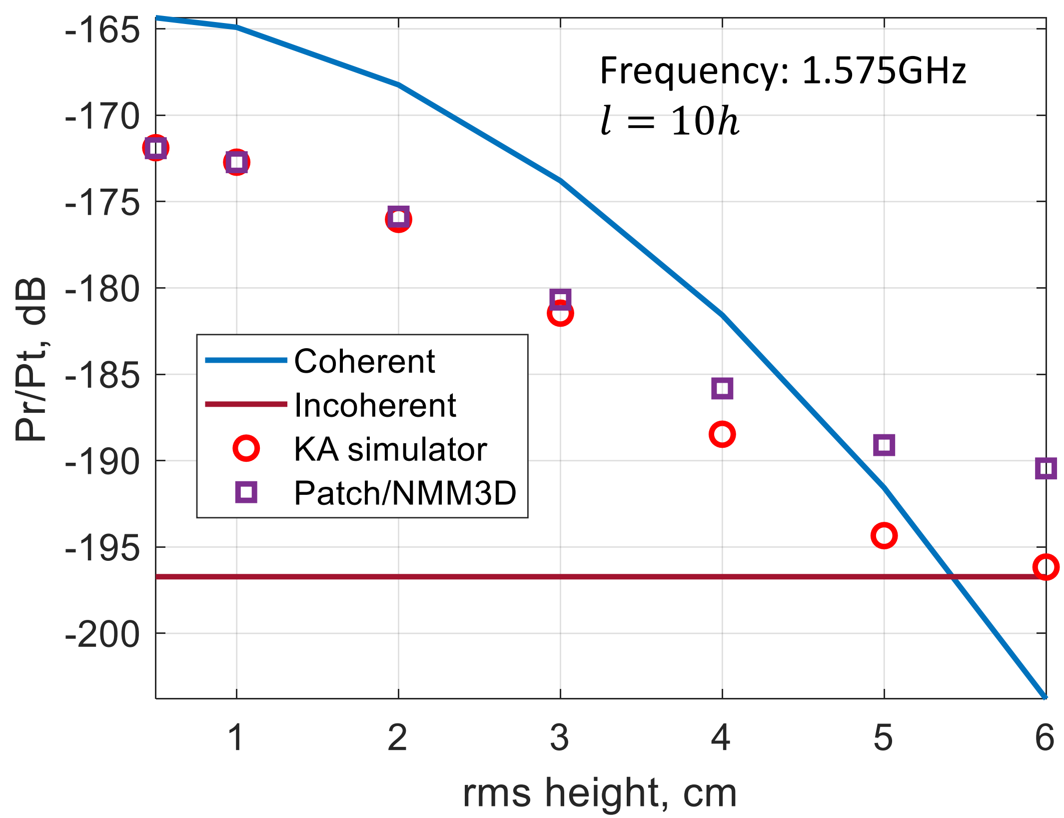
Patch Model: single physical area, multiple elevations (90000 elevations)

- Patch model and KA agree for small rms heights
- rms heights 6cm: 5dB differences between NMM3D and Kirchhoff
- Exponential factor $e^{-4k^2h^2(cos\theta)^2}$ in Kirchhoff not correct for large rms heights in DeSanto (1974)
- NMM3D agrees with DeSanto.

Multiple elevations in Georgia
$31°49'50''$N, $83°49'50''$W



DEM resolution 30m by 30m

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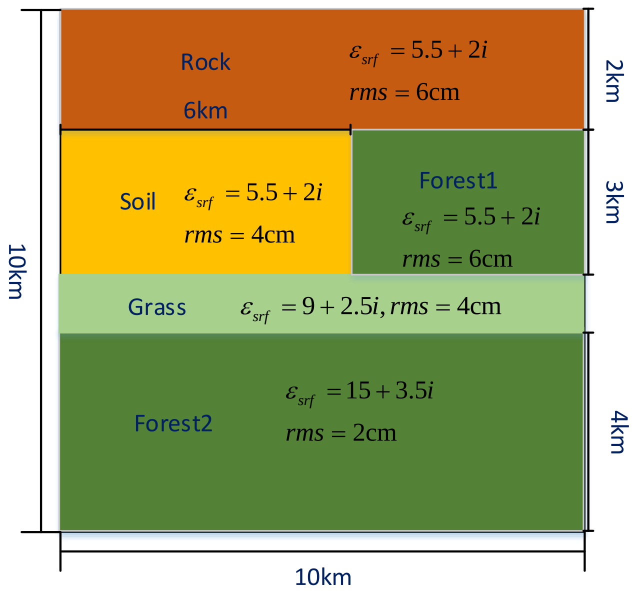
J.A. DeSanto and O. Shisha, “Numerical solution of a singular integral equation in random rough surface scattering theory.” *Journal of Computational Physics*, vol. 15, no. 2, pp.286-292, 1974.

Patch Model: multiple physical areas, single elevation



Correlation formula

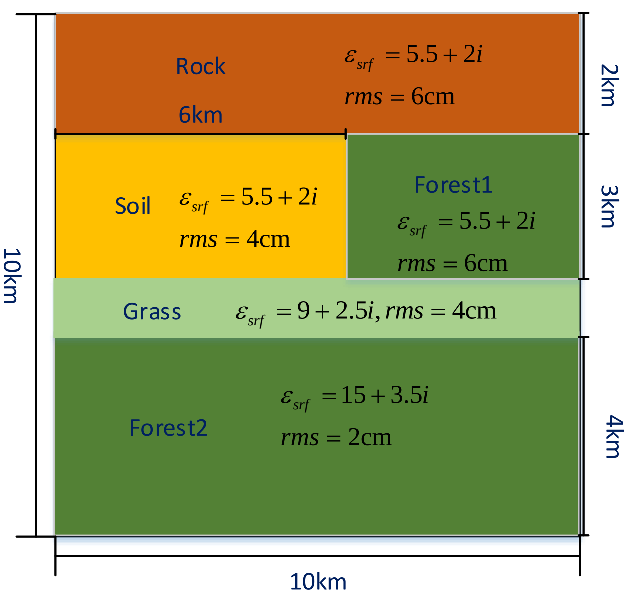
$$\left\langle \left| \bar{E}_s \right|^2 \right\rangle = \sum_{n=1}^{N} \left\langle \left| \bar{E}_n^s \right|^2 \right\rangle + \sum_{n=1}^{N} \sum_{m=n+1}^{N} 2\,\mathrm{Re}\left(\left\langle \bar{E}_n^s \right\rangle \left\langle \bar{E}_m^{s*} \right\rangle \right)$$

- ❑ Strong correlations of coherent fields

Patch	Power each area	Correlation	Correlation Value	P_r/P_t dB
Bare soil	4.302×10^{-14}	Bare soil & Grass	2.016×10^{-14}	-
Grass	2.851×10^{-15}	Bare soil & Forest2	-6.521×10^{-15}	-
Forest2	2.984×10^{-16}	Grass & Forest2	-1.798×10^{-15}	-
Total	4.621×10^{-14}	Total	1.184×10^{-14}	-172. 45dB
		KA simulator		**-173.15dB**

Patch Model: multiple physical areas, single elevation

- Coherent contribution dominates

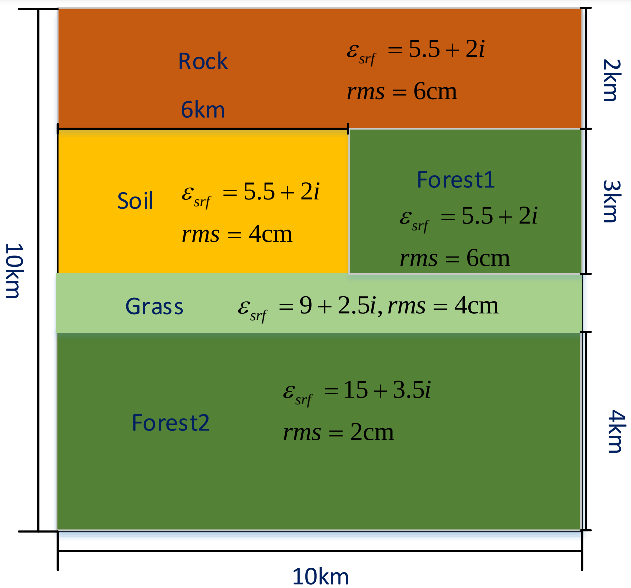


Coherent & incoherent formula

$$\left\langle \left|\bar{E}_s\right|^2 \right\rangle = \left|\left\langle \bar{E}_s \right\rangle\right|^2 + \sum_{n=1}^{N} \left\langle \left|\bar{E}_n^s - \left\langle \bar{E}_n^s \right\rangle\right|^2 \right\rangle$$

Model		Pr/Pt dB
KA simulator	Coherent component	-173.15
	Incoherent component	-200.39
	Total	**-173.15**
Patch/NMM3D	Coherent component	-172.45
	Incoherent component	-202.63
	Total	**-172.45**

Patch Model: multiple physical areas, multiple elevations (10000 elevations)



100 elevations by 100 elevations

Correlation formula

$$\left\langle \left|\bar{E}_s\right|^2 \right\rangle = \sum_{n=1}^{N} \left\langle \left|\bar{E}_n^s\right|^2 \right\rangle + \sum_{n=1}^{N} \sum_{m=n+1}^{N} 2\,\mathrm{Re}\left(\left\langle \bar{E}_n^s \right\rangle \left\langle \bar{E}_m^{s*} \right\rangle \right)$$

- Coherent component reduced by elevations

Self-term	Power each area	Correlation	Value	P_r/P_t dB
Bare soil	6.215×10^{-15}	Bare soil & Grass	5.615×10^{-15}	-
Forest2	1.298×10^{-15}	Bare soil & Forest2	-2.409×10^{-15}	-
Grass	7.141×10^{-16}	Grass & Forest2	-9.060×10^{-16}	-
Total	8.298×10^{-15}	Total	2.3×10^{-15}	-180.34dB
KA simulator				**-181.28dB**

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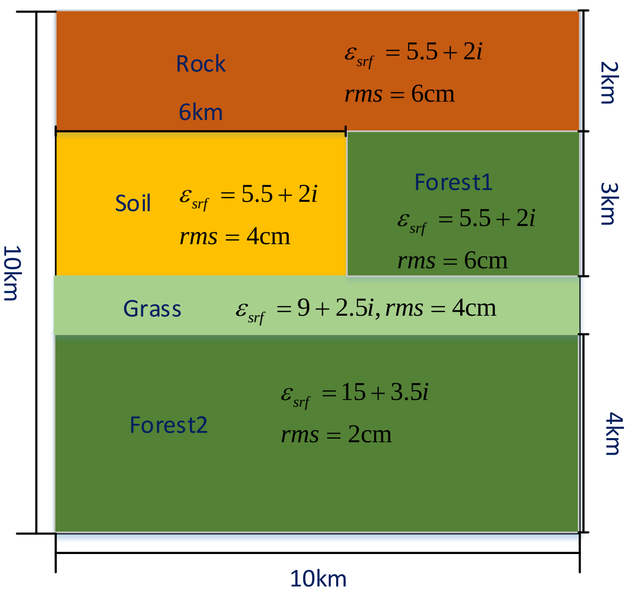
Patch Model: multiple physical areas, multiple elevations (10000 elevations)

Coherent & incoherent formula

$$\left\langle \left|\bar{E}_s\right|^2 \right\rangle = \left|\left\langle \bar{E}_s \right\rangle\right|^2 + \sum_{n=1}^{N} \left\langle \left| \bar{E}_n^s - \left\langle \bar{E}_n^s \right\rangle \right|^2 \right\rangle$$

- Coherence reduced by elevations



Model		Pr/Pt dB
KA (single elevation)		**-173.15**
KA simulator	Coherent component	-181.33
	Incoherent component	-200.55
	Total	**-181.28**
Patch/NMM3D	Coherent component	-180.36
	Incoherent component	-202.68
	Total	**-180.34**

Land Remote Sensing: Differences

	Radar Backscattering (SMAP)	GNSS-R (CYGNSS)
Radar configuration	Monostatic	Bistatic, Specular
Field components	Incoherent Fields	Coherent Field and Incoherent Field
Land surface profile	Single elevation	Multiple Elevations (Topography, DEM) influence Coherent Fields
Validation	Incoherent Fields	Coherent Fields, Specular Incoherent fields

Summary: 2 recent models by our Group

	KA simulator	Patch model
Patch size	2cm by 2cm	30m by 30m
Computation requirements	40 hours (20 cores)	0.6 seconds (1 core)
Accuracy	Kirchhoff approx.	NMM3D LUT more accurate
Surface type	Gaussian correlation functions	Gaussian and exponential
Formulation	Kirchhoff integral	Correlation formula and coh&incoh formula are consistent (different physical interpretations)
Land surface	Multiple elevations	Multiple elevations
Components	Both coherent and incoherent	Both coherent and incoherent