On the Nature of GNSS-R Land Surface Specular Scattering

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and Other Signals of Opportunity
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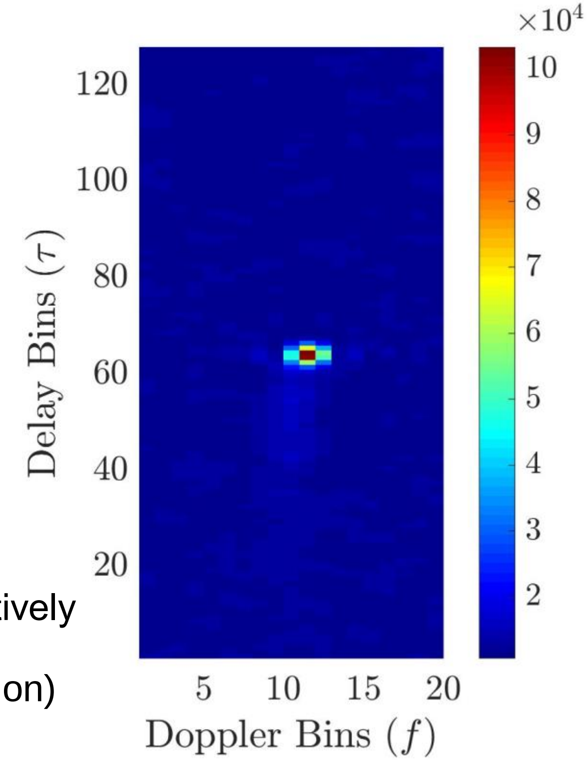
Motivation

- Growing datasets available from CYGNSS and TDS-1 show presence of coherent reflections over land surfaces (and ocean surfaces in some situations)
  - Coherent returns governed by Friis formula, incoherent by the bistatic radar equation
- Lots of interest in understanding coherent returns and how they could be used for remote sensing
  - Spatial resolution? Amplitude? Behavior versus frequency?
- Many current investigations looking at models for coherent return
- This talk looks at the fundamental theory of spaceborne coherent returns and what factors might impact them
  - "Coherence" here = contributions to received field from many points on Earth's surface have similar phases and add constructively
  - Vegetation neglected throughout (but should be just an attenuation)



See also Balakhder et al, "Signals of opportunity analysis of coherency properties from bistatic ocean and land returns," submitted to IEEE TGRS, 2019.

Formulation

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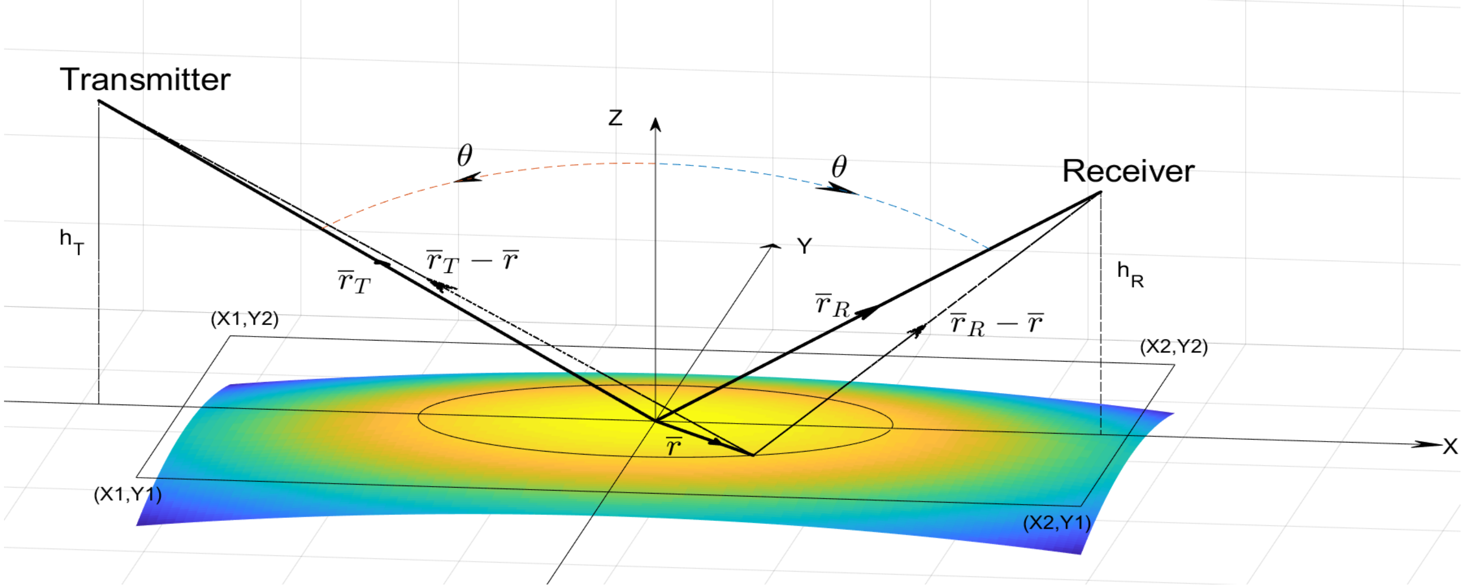
- Consider transmitter and receiver observing spherical Earth surface in a specular geometry; transmitter and receiver are far compared to smooth region and Fresnel zone sizes (spaceborne case)
- Earth surface may be locally flat or rough; we will consider only a truncated "flat region" (e.g. a water body) and ignore other regions
- First few Fresnel zones will dominate any coherent response (if present) so we will neglect variation in antenna gains and in 1/R terms over Earth's surface



Physical Optics Solution for Fields

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Physical optics formulation of surface scattering applicable for specular geometries, and states for a "flat" portion of a spherical Earth that

$$\overline{E}_s = \frac{ik_0 \cos\theta}{2\pi} \left[\left(\hat{h}_s \hat{h}_i \Gamma_H + \hat{v}_s \hat{v}_i \Gamma_v \right) \cdot \hat{e}_i \right] \int_A dA \frac{e^{-ik_0 r_{tot}}}{|\overline{r}_T - \overline{r}| \, |\overline{r}_R - \overline{r}|}$$

where Γ's are the Fresnel reflection coefficients and

$$k_0 r_{tot} \approx k_0(r_R + r_T) + k_0 \rho^2 \left(\frac{r_R + r_T}{2 r_R r_T} \left[\sin^2\phi + \cos^2\theta \cos^2\phi \right] + \frac{\cos\theta}{a_{eff}} \right)$$

$$= k_0(r_R + r_T) + \frac{\pi x^2}{F_{1x}^2} + \frac{\pi y^2}{F_{1y}^2} \quad \text{for specular geometries}$$

with

$$F_{1x} = \frac{F_1}{D_x \cos\theta} \qquad D_x = \sqrt{1 + 2\frac{F_1}{a_{eff}} \frac{F_1}{\lambda \cos\theta}}$$

$$F_{1y} = \frac{F_1}{D_y} \qquad D_y = \sqrt{1 + 2\frac{F_1}{a_{eff}} \frac{F_1 \cos\theta}{\lambda}}$$

$$F_1 = \sqrt{\frac{\lambda \left(r_R r_T \right)}{\left(r_R + r_T \right)}}$$

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Earth radius

$$= k_0(r_R + r_T) + \frac{\pi x^2}{F_{1x}^2} + \frac{\pi y^2}{F_{1y}^2}$$

Fresnel zone radii

Spherical Earth factors

with

$$F_{1x} = \frac{F_1}{D_x \cos\theta} \qquad F_{1y} = \frac{F_1}{D_y}$$

$$D_x = \sqrt{1 + 2\frac{F_1}{a_{eff}}\frac{F_1}{\lambda\cos\theta}} \qquad D_y = \sqrt{1 + 2\frac{F_1}{a_{eff}}\frac{F_1\cos\theta}{\lambda}}$$

$$F_1 = \sqrt{\frac{\lambda\,(r_R r_T)}{(r_R + r_T)}}$$

Simplifying….

We now have

$$\overline{E}_s = \frac{ik_0 \cos\theta e^{-ik_0(r_R+r_T)}}{2\pi r_R r_T}\left[\left(\hat{h}_s\hat{h}_i\Gamma_H + \hat{v}_s\hat{v}_i\Gamma_v\right)\cdot\hat{e}_i\right]$$
$$\int_A d\overline{r}\exp\left(-i\frac{\pi x^2}{F_{1x}^2}\right)\exp\left(-i\frac{\pi y^2}{F_{1y}^2}\right)$$

Substitute $x' = \sqrt{\pi}x/F_{1x}$, $y' = \sqrt{\pi}y/F_{1y}$ (note distorts boundary) to get

$$\overline{E}_s = \frac{ie^{-ik_0(r_R+r_T)}}{\pi(r_R+r_T)D_xD_y}\left[\left(\hat{h}_s\hat{h}_i\Gamma_H + \hat{v}_s\hat{v}_i\Gamma_v\right)\cdot\hat{e}_i\right]$$
$$\int_{A'} d\overline{r}'\exp\left(-i\rho'^2\right)$$
$$= Z_f\overline{E}_{friis}$$

$$Z_f = \frac{i}{\pi}\int_{A'} d\rho' d\phi' \rho' \exp\left(-i\rho'^2\right)$$

Simplifying….

We now have

$$\overline{E}_s = \frac{ik_0 \cos\theta e^{-ik_0(r_R+r_T)}}{2\pi r_R r_T}\left[\left(\hat{h}_s\hat{h}_i\Gamma_H + \hat{v}_s\hat{v}_i\Gamma_v\right)\cdot\hat{e}_i\right] \int_A d\overline{r}\,\exp\left(-i\frac{\pi x^2}{F_{1x}^2}\right)\exp\left(-i\frac{\pi y^2}{F_{1y}^2}\right)$$

Substitute $x' = \sqrt{\pi}x/F_{1x}$, $y' = \sqrt{\pi}y/F_{1y}$ (note distorts boundary) to get

$$\overline{E}_s = \frac{i}{\pi}\frac{e^{-ik_0(r_R+r_T)}}{(r_R+r_T)\,D_x D_y}\left[\left(\hat{h}_s\hat{h}_i\Gamma_H + \hat{v}_s\hat{v}_i\Gamma_v\right)\cdot\hat{e}_i\right] \int_{A'} d\overline{r}'\,\exp\left(-i\rho'^2\right) = Z_f\overline{E}_{friis}$$

Spherical Earth Friis field

$$Z_f = \frac{i}{\pi}\int_{A'} d\rho'\,d\phi'\,\rho'\,\exp\left(-i\rho'^2\right)$$

Integral over distorted "flat" region on Earth to determine field relative to Friis prediction

Field relative to the Friis formula

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Let's look at $Z_f = \frac{i}{\pi} \int_{A'} d\rho' d\phi' \rho' \exp\left(-i\rho'^2\right)$

If we can write the boundary of the smooth region as $\rho' = \rho'_{max}(\phi')$ (i.e. the "flat region" contains the specular point and boundary is single valued in range), then we can integrate radially to obtain

$$Z_f = 1 - \frac{1}{2\pi} \int_0^{2\pi} d\phi' \exp\left(-i\rho'^2_{max}(\phi')\right)$$

i.e. the field relative to the Friis formula arises as one minus an average of equal amplitude phasors evaluated on the flat region boundary

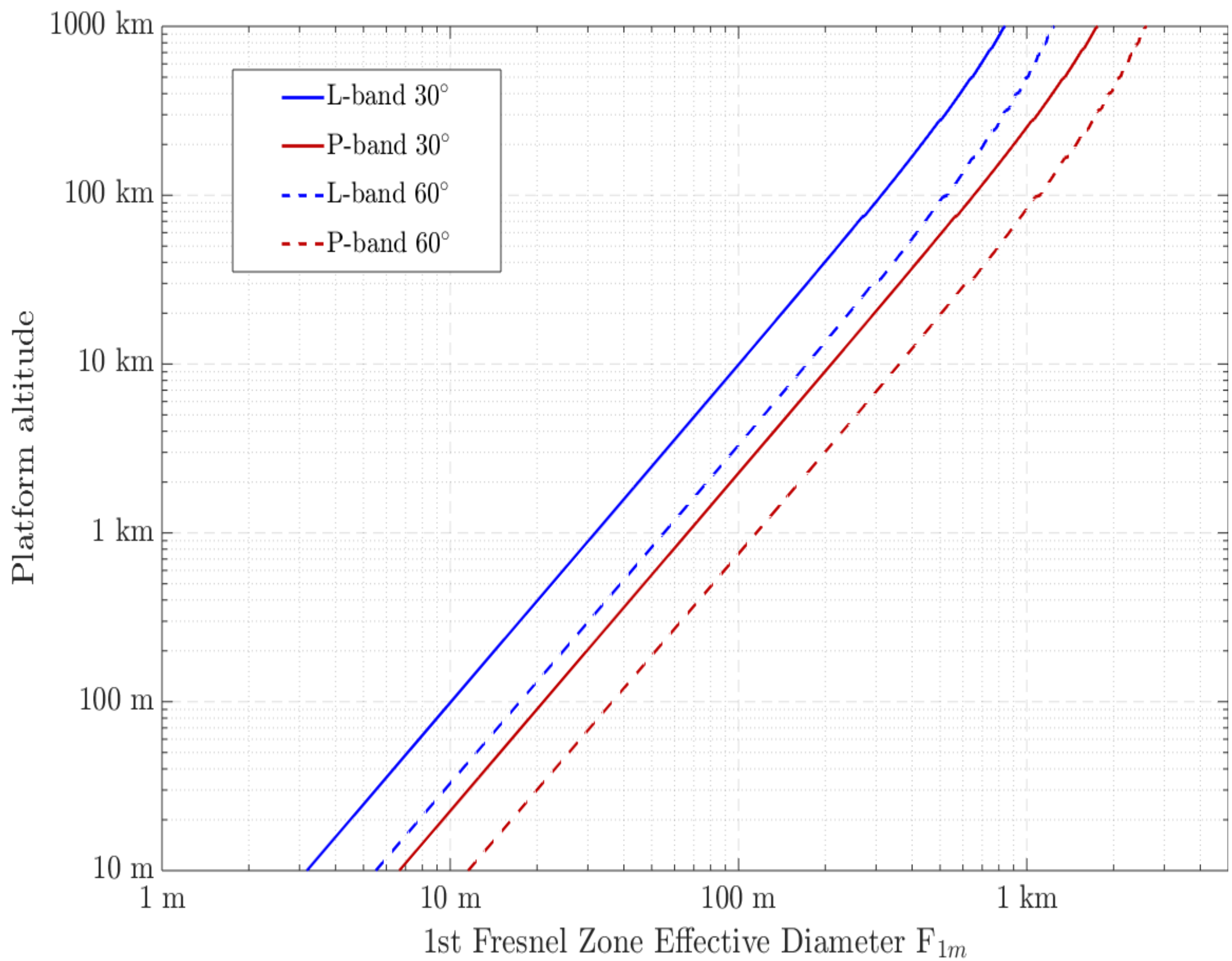
This implies the coherent field for a finite flat region (i.e. a water body) should be highly dependent on the boundary shape

Also, for flat regions small compared to the Fresnel zone, $Z_f = \frac{iA}{F_{1x}F_{1y}}$ and the field amplitude is proportional to flat region area

First Fresnel zone region size

Transmitter assumed to be in geostationary orbit so much further than receiver from specular point

Fresnel zone diameters (2x geometric mean of F1x and F1y) for spaceborne receiver range from ~ 500 m up to ~ 3 km



A few particular flat region shapes

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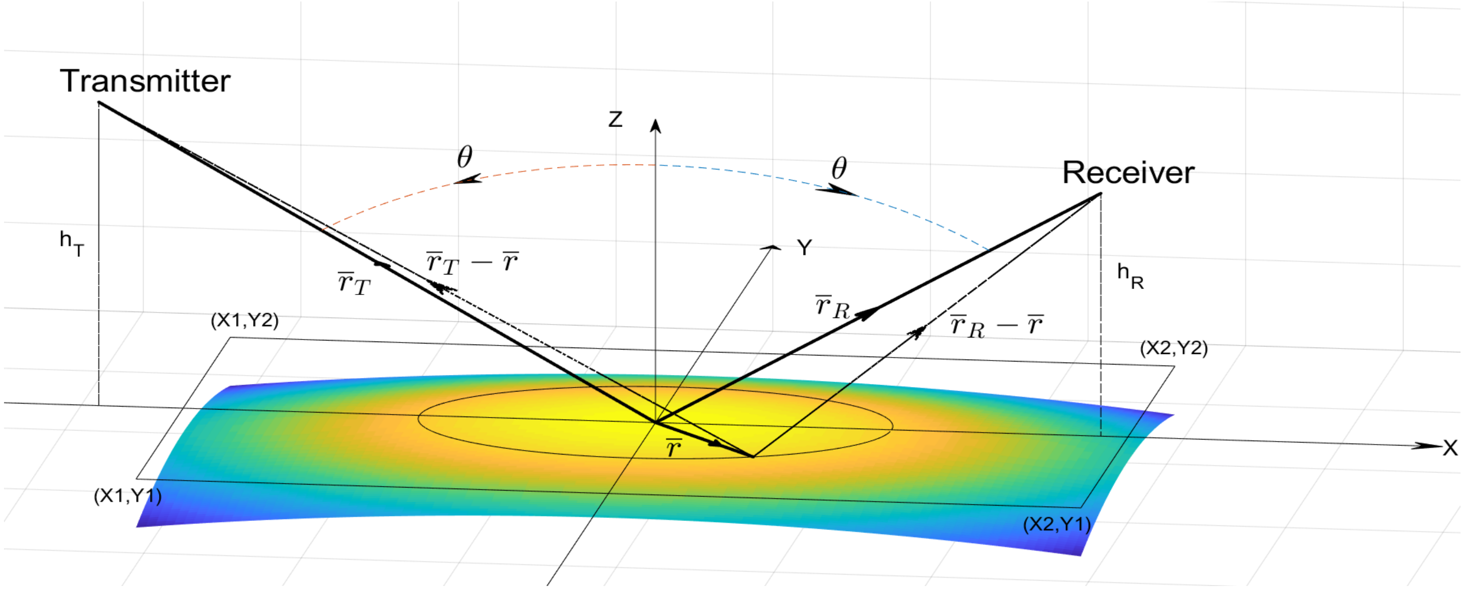
For an elliptical disk boundary $\rho'_{max}(\phi') = R_{max}\frac{\sqrt{\pi}}{F_{1y}}$ we obtain

$$Z_{f,disk} = \left(1 - \exp\left(-i\pi\left(\frac{R_{max}}{F_{1y}}\right)^2\right)\right)$$

which oscillates between 0 and 2 depending on the disk size relative to the Fresnel zone -> never converges! Clearly a lot of variability!

Now consider a rectangular flat region



Rectangular flat region

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For the rectangular boundary $Z_{f,rect} = \frac{i}{2} Q\left(\frac{\sqrt{2}x}{F_{1x}}\right)\Big|_{X_1}^{X_2} Q\left(\frac{\sqrt{2}y}{F_{1y}}\right)\Big|_{Y_1}^{Y_2}$

where the Q functions involve Fresnel integrals

$$Q(z) = C(z) - iS(z) = \int_0^z dt\, e^{-i\frac{\pi t^2}{2}}$$

$$\approx z \quad \text{(small arguments)}$$

$$\approx \frac{e^{-i\frac{\pi}{4}}}{\sqrt{2}}\left(1 - \sqrt{2}e^{-i\frac{\pi}{2}\left(z^2 - \frac{1}{2}\right)} \frac{z}{1 + i\pi z^2 - \frac{2}{5 + i\pi z^2}}\right) \quad (z>0.72)$$

Rectangular flat region

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Oscillatory term whose amplitude dies off as 1/z

These results again show the increase with area in the small flat region size limit, as well as an oscillatory behavior for larger sizes that decays as the rectangular region becomes larger

We can also show that the field decays in amplitude as the rectangular flat region is offset from the specular point

Rectangular Examples

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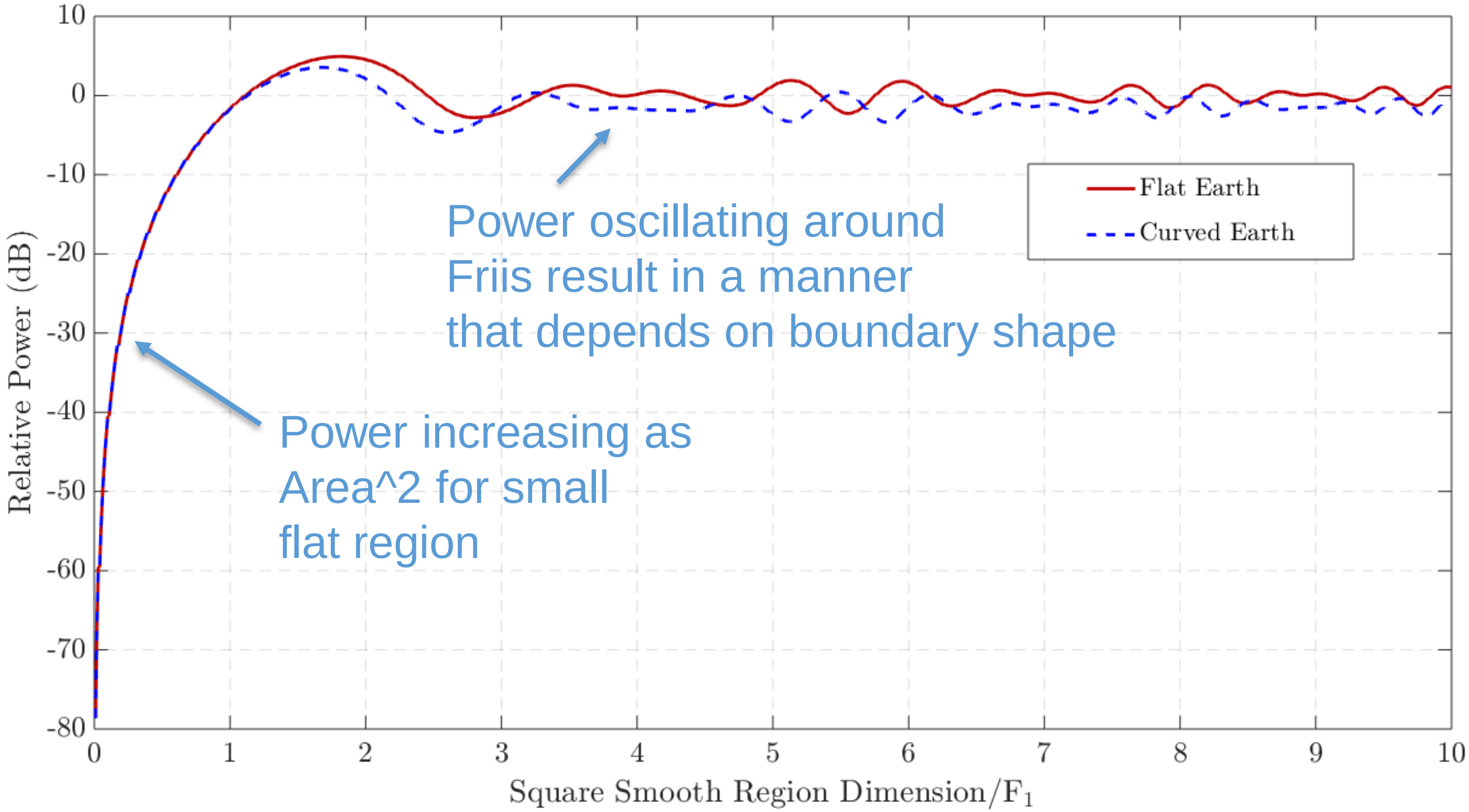
Coherent field power relative to Friis formula for a square flat region centered on the specular point, θ=30º



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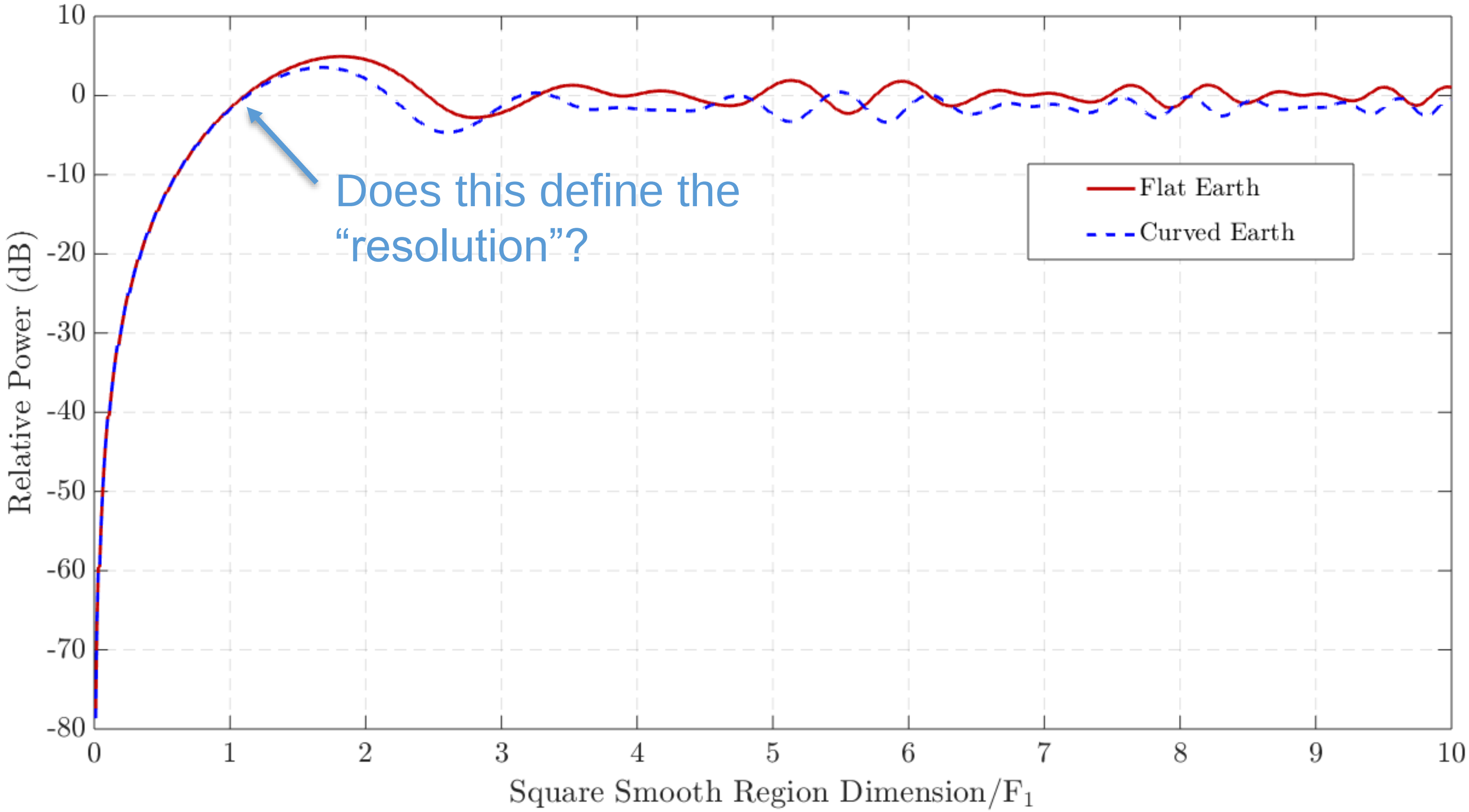
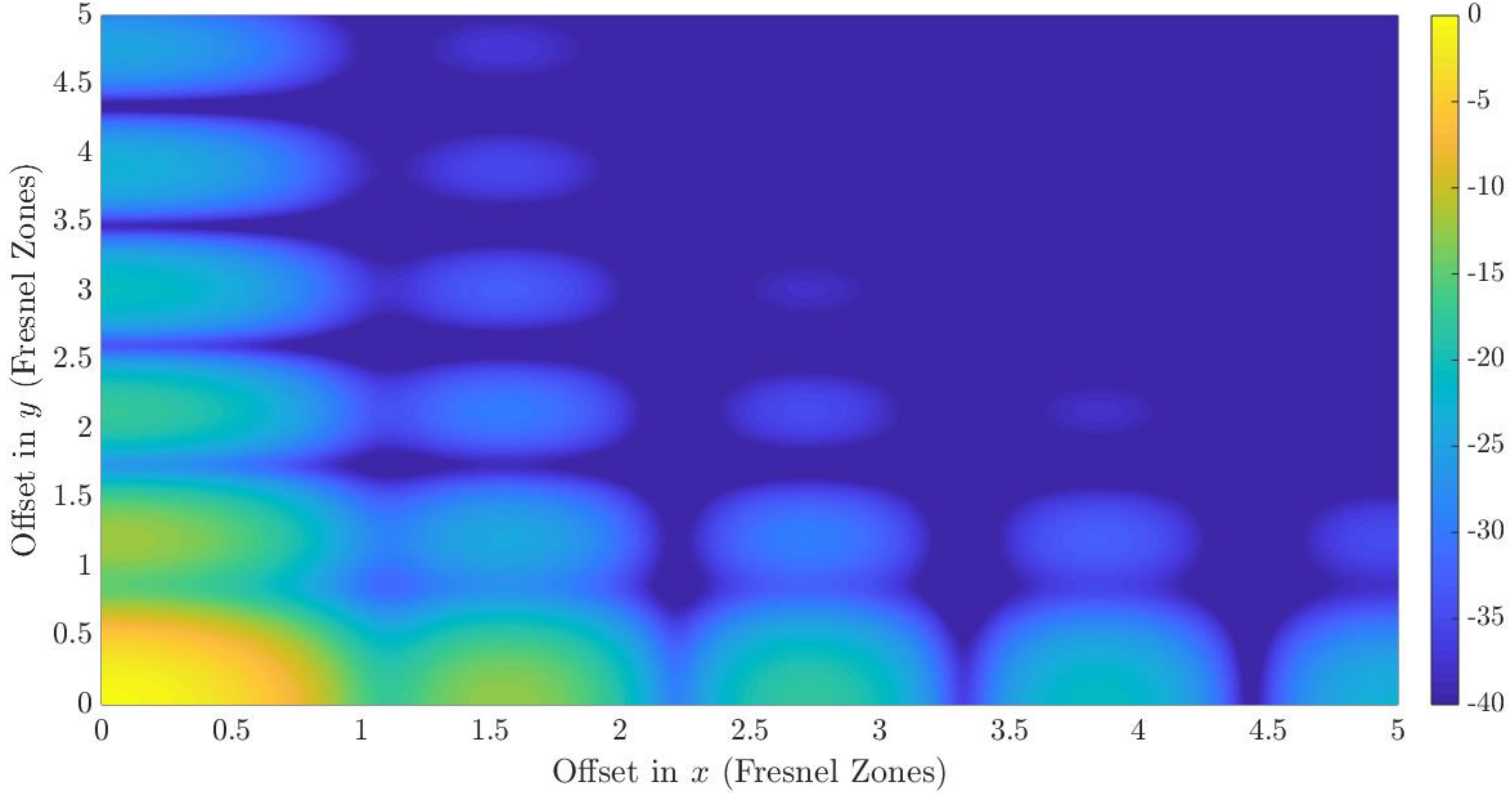


Offset Rectangular Flat Region

Power relative to Friis formula (dB) for square flat region of size F_1 as function of offset from specular point, θ=30º



What is the surface isn't flat?

Surface roughness will cause a shift of $e^{-i(2k_0 h \cos\theta)}$ for height h above the specular point

Within the first Fresnel zone, we can approximate that there is practically no other phase shift than this

If heights vary within the Fresnel zone such that $2k_0 h \cos\theta$ takes on a variety of appreciable values, we will have many terms adding out of phase and canceling the field, i.e. the received field will reduce rapidly

If we can model surface heights within the first Fresnel zone as arising from a stationary Gaussian random process of rms height h_0 we obtain the classical result that the expected coherent received power is scaled by $e^{-(2k_0\, h_0 \cos\theta)^2}$

Note that the rms height of interest can be approximated as that within the first Fresnel zone

Coherent or incoherent?

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- Coherent reflection:

$$P_{pq}^{c} = \frac{P_t G^t}{4\pi(R_1+R_2)^2}\cdot\frac{G^r\lambda^2}{4\pi}\Gamma_{lr}$$

λ : Wavelength

P_t : Transmit Power

G^t, G^r : TX and RX antenna gain

R_1, R_2 : Ranges from TX and SP, SP and RX

Wavenumber
RMS height
Incident Angle

$$\Gamma_{lr} = |R_{lr}|^2 e^{-4k^2h^2\cos^2\theta}$$: Power Reflectivity

R_{lr} : Fresnel reflection coefficient

Function of soil moisture, soil texture, and inc. angle

- Incoherent:

$$P_{pq}^{i} = \frac{P^t G^t}{4\pi R_1^2}\cdot\frac{G^r\lambda^2}{4\pi}\iint_A \frac{\sigma^0}{4\pi R_2^2}dxdy$$

$$\sigma_0 \propto \frac{|R_{lr}|^2}{\cos^2\theta\cdot\tilde{s}^2}$$

Surface correlation length

$$\tilde{s}^2 = (h/l)^2\left(4k^2h^2\right)$$

: Effective slope variance for exponential surface (Mean Square Slope (MSS))

Function of soil moisture, soil texture, inc. angle, and MSS

Predicting the ratio of coherent to incoherent returns

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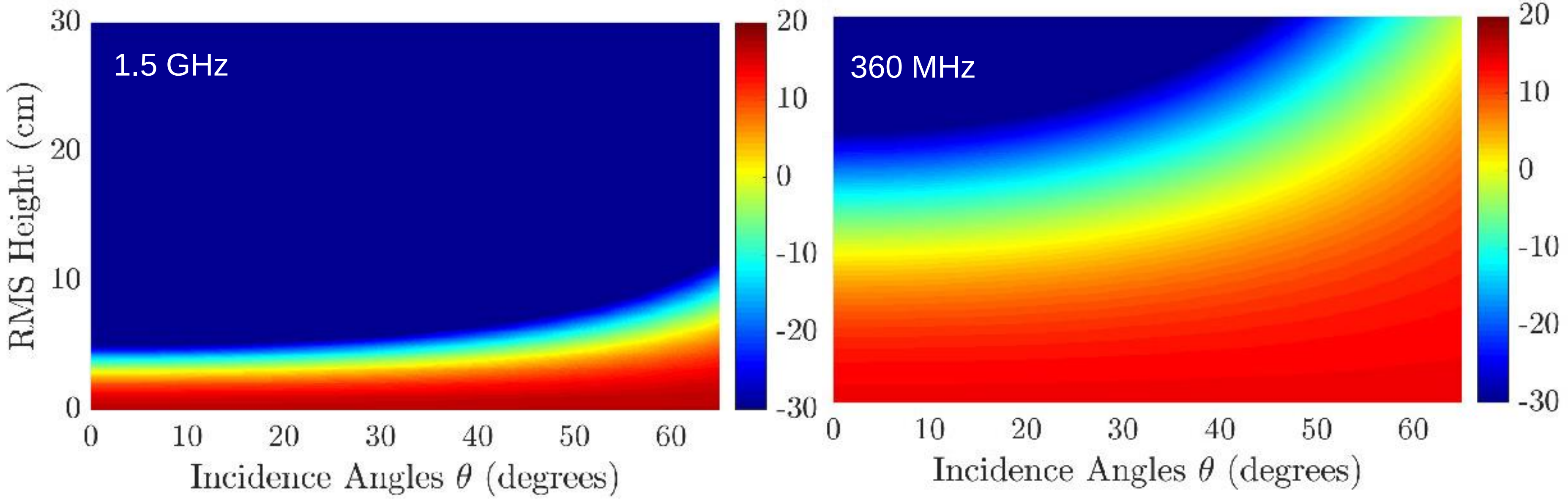
Incoherent returns are impacted by the Earth surface within the DDM point-spread function – typically a larger area

With a few approximations applicable to the spaceborne case, we can predict the ratio of the coherent to incoherent power as a function of incidence angle and surface rms height:



Importance of coherent term clearly reduces with surface rms height

Should we expect rms heights of a few cm to 10's of cm on Earth's surface within the ~ 0.5 -3 km sized Fresnel zones? DEM's suggest most places not this flat.

Conclusion: Coherence at L-band most likely to occur for inland water bodies

Why do smooth water areas matter more than smooth land areas?

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- It is reasonable to ask why water bodies in only a portion of the first Fresnel zone can produce strong coherent returns while locally flat land surfaces do not
- First, Fresnel reflection coefficient of water is stronger than land, so water can dominate surrounding land area contributions and avoid cancellation
- Locally smooth land surfaces (having rms heights of ~ 1-2 cm) could achieve constructive interference over moderate length scales , e.g. 1- 20 m
- However other nearby land surfaces having seemingly similar rms heights and dimensions but shifted up or down by topography would contribute destructively
- This makes it difficult for land surfaces to cause coherent returns
- A few land regions on Earth may have sufficiently small topographic variations over ~ 100's of m scales to achieve coherence, but this would be rare
- Difficult to assess any of this using DEM's because their uncertainties are larger than the cm-scale rms heights of interest

Conclusions

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- Coherent reflection can occur over land surfaces when the Earth's surface (or portions of it) within ~ the first Fresnel zone have height variations smaller than a few cm (L-band) or 10's of cm (P-band)
  - This should mostly occur for L-band spaceborne measurements for inland water bodies
  - P-band remains to be demonstrated for spaceborne case; not immediately clear that coherence will be the dominant land mechanism
- Amplitude of coherent returns very sensitive to:
  - "Flat region" boundary shape and offset from specular point
    - Power proportional to Area^2 for small flat regions
  - Any small scale roughness (e.g. due to winds on inland water bodies – can be modeled as a finite-fetch case)
  - Surface permittivity (through Fresnel reflectivity)
- "Spatial resolution" of coherent return related to first Fresnel zone size but hard to determine completely given oscillations of received power
- Attempts at spaceborne remote sensing using coherent return amplitudes (e.g. inland water bodies at L-band) need to account for these effects